Optimization of Piezoceramic Sensor/Actuator Placement for Vibration Control of Laminated Plates

Young Kyu Kang*
University of Manchester, Manchester M13 9PL, England, United Kingdom
and
Hyun Chul Park† and Brij Agrawal‡
U.S. Naval Postgraduate School, Monterey, California 93943

Introduction

PROPER selection of the number and location of piezoelectric sensors/actuators is one of the critical issues for structural vibration control. A vast amount of research has been devoted to the placement and design of sensors or actuators (or both)[1,2] for flexible structure control. Also, many concepts such as controllability and observability[3] have been proposed. These methods have been shown to yield less effective results for damping in structural vibration because their concepts are focused on stiffness.

In this Note, optimization of the sensor/actuator placement for structural vibration control of the laminated composite plates is investigated numerically and verified experimentally for various fiber orientations in the plates. Damping and stiffness of the adhesive layer and the piezoceramics are taken into account in the finite element formulation. A structural damping index (SDI)[4] is defined from modal damping by combining each vibrational mode with a weighting factor. For vibration control of a plate, the gradient method is proposed to locate the optimum position of the sensor/actuator on the plate. Effects of the outer-layer fiber orientation of the laminated plate on the optimum sensor/actuator placement are investigated numerically and experimentally.

Method

Laminated composite plates with a piezoceramic sensor/actuator are modeled as two-dimensional plates. The detailed derivation of the finite element equations has been explained in Ref. 4.

The equation of motion of a structure in active control is expressed as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{F}_{\text{ext}} + \boldsymbol{D}_a u_c \tag{1}$$

where $\boldsymbol{q}(t)$ is the displacement vector, $\boldsymbol{M}$ the mass matrix, $\boldsymbol{C}$ the structural damping, $\boldsymbol{K}$ the stiffness matrix, $\boldsymbol{F}_{\text{ext}}$ the external force vector, $\boldsymbol{D}_a$ the actuator influence matrix, and u_c the control input to the structure. Introducing the modal coordinate transformation results in the modal equation of motion as follows:

$$\bar{M}\ddot{\eta} + \bar{C}\dot{\eta} + \bar{K}\eta = \bar{F}_{\text{ext}} + \bar{D}u_c \tag{2}$$

The first-order state-space form of the system equations equivalent to Eq. (2) is

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u_c + B_0 \tag{3}$$

When the piezoceramic sensor detects the strain rate of the structure, the actuator functions to increase the modal damping $\bar{C}$ of the system matrix $\boldsymbol{A}$ in Eq. (3). Total modal damping of the structure is composed of inherent damping $\bar{C}$ and active damping $\bar{C}_a$, which is actuated by the control input.

The SDI is defined by taking the modal damping and contribution of each vibrational mode into account as follows:

$$\text{SDI} = \sum_r 2\zeta_r \omega_r c_r \tag{4}$$

where the weighting factor c_r is calculated using modal orthogonality.

The SDI is chosen as the objective function to present a general formulation of the optimization of piezoceramic sensor/actuator placement. The optimum location of the sensor/actuator is then determined by maximizing the SDI. The SDI of the plate can be shown to be

$$\text{SDI} = \text{SDI}\left(\theta, \bar{x}_i, \bar{y}_i, C_{x_i}, C_{y_i}, t_p, G\right) \tag{5}$$

where design variable θ is the outer-layer fiber orientation of the laminated composite plate, the variables $\bar{x}_i$ and $\bar{y}_i$ are the location of the center of the piezoceramic sensor/actuator in the ith placement, C_{x_i} and C_{y_i} are the size of the ith piezoceramic sensor/actuator, t_p is the thickness of the piezoceramics, and G is the feedback gain. Although the thickness of the piezoceramics and the feedback gain are considered as design variables, they are fixed because the effects of the other variables are dominant. Interaction between the passive and the active controls of the composite structures has been carried out. The gradient of the objective function is calculated as follows:

$$\text{grad(SDI)} = \Delta(\text{SDI})/\Delta S \tag{6}$$

where ΔS is the distance between two sensor/actuator (S/A) locations and $\Delta(\text{SDI})$ is the difference of the SDI between the current position and the candidate for the next position. When the position of the actuation force (or moment) for the active control moves, the mass and stiffness of the system must be evaluated for every iteration and also must be considered in calculating the gradient of the objective function. To decide the optimum search direction from some arbitrary initial S/A location, the objective function and its gradient are calculated for every direction, taking the structural properties of the S/A into consideration. The position is then updated using the gradient information, and the objective function and the gradient are calculated again. The search continues until the optimum is achieved. Design constraints should be specified to confine the design variable within a feasible region. The optimization problem is as follows:

$$\begin{aligned} &\text{minimize} \quad f \\ &\text{subject to} \quad g_i \end{aligned} \tag{7}$$

where $f = -(\text{SDI})$ and g_i are design constraints.

Results and Discussion

Optimization of the collocated piezoceramic sensor/actuator placement for vibration control of cantilevered laminated plates is investigated with stacking sequences of $[\theta_4/0_2/90_2]_s$, where $\theta = 0$, 15, 30, 45, 60, and 90 deg. Material properties are listed in Ref. 4. The dynamic characteristics of the lower six modes in the composite plates are analyzed and used in the calculation of the SDI. The width and the thickness of the piezoceramics are 20 and 0.5 mm, respectively. To express the location of a piezoceramic S/A in the plate, the direction of plate length is chosen as the x axis. One piezoceramic sensor/actuator is used in the simulation to express the location systematically.

The SDI of the plate depends on the fiber orientation of the outer layer, as well as the location of the S/A for a fixed length of the piezoceramics. The SDI for the $[0_4/0_2/90_2]_s$ plate is calculated and plotted in Fig. 1a for every possible position of the S/A, which we called the scanning method. The S/A is collocated with a length of 25 mm. The SDI becomes small, in Fig. 1a, as the S/A position moves away from the clamped side of the cantilevered plate. The SDI becomes large when the S/A moves close to the clamped side. The optimum location of the S/A for the $[0_4/0_2/90_2]_s$ plate is at A (12.5, 10). Figure 1b shows the SDI for the $[45_4/0_2/90_2]_s$ plate. The

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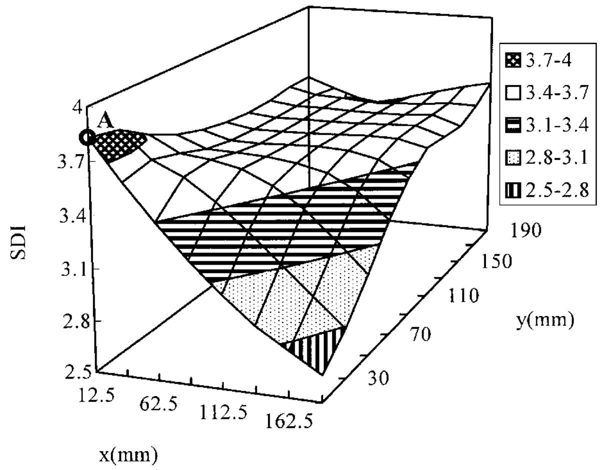
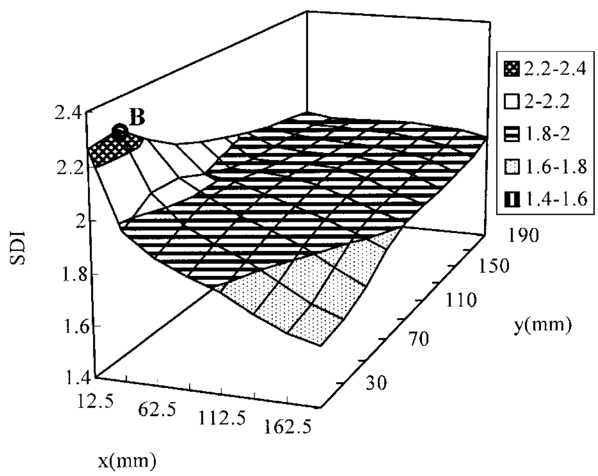
*Postdoctoral Researcher, Department of Mechanical Engineering; currently Assistant Professor, School of Mechanical Engineering, Pusan National University, Pusan 609-735, Republic of Korea. Member AIAA.

†Visiting Professor, Department of Mechanical Engineering; Associate Professor, Department of Mechanical Engineering, Pohang University of Science and Technology (POSTECH), Pohang, Kyungbuk, 790-784, Republic of Korea. Member AIAA.

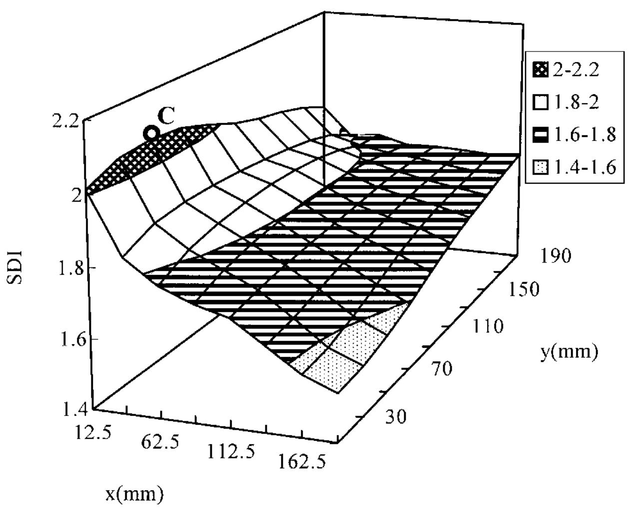
‡Professor, Department of Aeronautics and Astronautics.



a) **Stacking sequence $[0_4/0_2/90_2]_s$ plate**



b) **Stacking sequence $[45_4/0_2/90_2]_s$ plate**



c) **Stacking sequence $[90_4/0_2/90_2]_s$ plate**

Fig. 1 SDI for the 25-mm-long piezoceramic sensor/actuator (PZT).

SDI is smaller than that of the $[0_4/0_2/90_2]_s$ plate for all locations. The optimum location of the S/A for the $[45_4/0_2/90_2]_s$ plate is at B (12.5, 30). Optimization using the gradient method can reduce computation time and can search the optimum location very efficiently. Optimization by the gradient method takes about 20% of the CPU time compared to that of the scanning method. Figure 1c represents the SDI for the $[90_4/0_2/90_2]_s$ plate, and the optimum location is at C (12.5, 50). It is located relatively far from the x axis, as compared with the $[0_4/0_2/90_2]_s$ plate of Fig. 1a. Comparing Fig. 1a with Fig. 1c, the SDI becomes smaller as the outer-layer fiber orientation increases. The optimum location of the piezoceramic S/A moves away from the x axis as the fiber orientation increases. This can be explained by the modal dominance. Weighting factors of the vibration modes become different as the bending stiffness of the plate changes. For a flexible plate such as the $[90_4/0_2/90_2]_s$ specimen, the first bending mode is dominant. The S/A should be located farther away from the x axis of the plate to control the bending mode efficiently, which is consistent with the physical phenomenon.

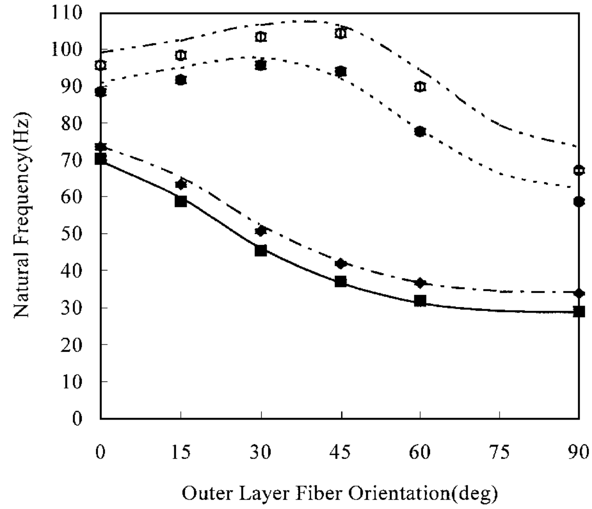


Fig. 2 Natural frequency of the plates without and with PZT: ——, bending without PZT [finite element method (FEM)]; ■, bending without PZT (experiment); - - - -, torsion without PZT (FEM); ●, torsion without PZT (experiment); – – – -, bending with PZT (FEM); ♦, torsion with PZT (experiment); – - – -, torsion with PZT (FEM); and ○, torsion with PZT (experiment).

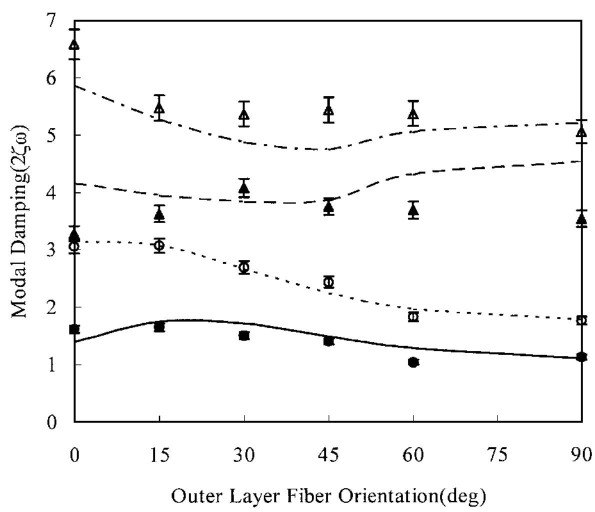


Fig. 3 Modal damping of the first bending and torsional modes (open and closed loop): ——, bending (gain = 0, FEM); ●, bending (gain = 0, experiment); - - - -, bending (gain = 2, FEM); ○, bending (gain = 2, experiment); – – –, torsion (gain = 0, FEM); ▲, torsion (gain = 0, experiment); – - – -, torsion (gain = 2, FEM); and △, torsion (gain = 2, experiment).

Experiments have been carried out to validate the formulation of the S/A placement. Configuration of the plate for the experiment is shown in Ref. 5. Figure 2 shows the natural frequency of the plate with and without piezoceramics. As the first bending and the first torsional mode are dominant in the plate vibration, the active modal damping of the first bending and the first torsional mode are measured to determine the SDI and are plotted in Fig. 3.

Conclusions

A general formulation and a systematic method for optimization of the S/A placement using the gradient method is proposed for vibration control of the laminated composite plates.

The optimum positions of the piezoceramic S/A tend to be near the clamping side of the cantilevered plate. The optimum locations also change as the outer-layer fiber orientation of the laminated plates changes.

A plate with higher stiffness is more effective than a plate with lower stiffness in controlling vibration actively. Numerical simulation and experimental results show that the SDI depends on the stiffness of the host structure and the location of the S/A.

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